

Cosmological Dynamics of the Tachyon With an Inverse Power-Law Potential¹

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We investigate tachyon dynamics with an inverse power-law potential $V(\varphi) \propto \varphi^{-\alpha}$. We find global attractors of the dynamics leading to a dust behavior for $\alpha > 2$ and to an accelerating universe for $0 < \alpha \leq 2$. We study linear cosmological perturbations and we show that metric fluctuations are constant on large scales in both cases. In the presence of an additional perfect fluid, the tachyon with this potential behaves as dust or dark energy.

KEY WORDS: cosmology; inflation; dark energy.

1. INTRODUCTION

Observations of the Cosmic Microwave Background (Bennet *et al.*, 2003) and of distant type Ia supernovas (Garnavich *et al.*, 1998; Perlmutter *et al.*, 1998; Riess *et al.*, 1998; Tonry *et al.*, 2003) indicate that the universe has been accelerating its expansion rate for the last 5–10 Gy. Prime candidates for causing this recent burst of expansion are a cosmological constant and a scalar field (Caldwell *et al.*, 1995; Frieman *et al.*, 1995; Ratra and Peebles, 1988; Wetterich, 1988; Zlatev *et al.*, 1995) which, for reasons unknown yet, started to dominate over the other types of matter just at our cosmological era. A phase of accelerated expansion driven by a scalar field (inflation) is also the preferred theory for the early universe, since it explains the homogeneity, flatness, and the near-scale-invariant spectrum of cosmological perturbations of the Universe.

Most scalar models of accelerated expansion involve canonical scalar field theories:

$$\mathcal{L}_C = -\sqrt{-g} \left[\frac{1}{2} \partial^\mu \varphi \partial_\mu \varphi + V(\varphi) \right]. \quad (1)$$

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Another class of scalar field theory, which appeared first as a certain field theory generalization of the Lagrangian of a relativistic particle (Bagla *et al.*, 2003; Padmanathan, 2002), is the *Born-Infeld* theory:

$$\mathcal{L}_{\text{BI}} = -\sqrt{-g}V(\varphi)\sqrt{1 + \frac{\partial^\mu\varphi\partial_\mu\varphi}{M^4}}. \quad (2)$$

Such a kind of scalar field was proposed in connection with string theory, since it seems to represent a low-energy effective theory of D-branes and open strings, and has been conjectured to play a role in cosmology (Bagla *et al.*, 2003; Padmanabhan, 2002; Sen, 2002a,b,c). Since the scalar field in the Lagrangian (2) stands for the *tachyon* of string theory, that name was also attached to the present scalar field, although in this incarnation the field is by no means unstable.

In the next Section we briefly review the dynamics of the tachyon scalar field. In Section 3 we determine the asymptotic tachyon-dominated backgrounds. In Section 4 we consider inhomogeneous perturbations in inflating and dust tachyon models. In Section 5 we determine how the tachyon will behave in the presence of another background fluid.

2. DYNAMICS OF THE TACHYON

The equation of motion for the tachyon is:

$$-\nabla^\mu\partial_\mu\varphi + \frac{\nabla_\mu\partial_\nu\varphi}{M^4 + \partial_\mu\varphi\partial^\mu\varphi}\partial^\mu\varphi\partial^\nu\varphi + M^4(\log V)_\varphi = 0. \quad (3)$$

At the homogeneous level this becomes:

$$\frac{\ddot{\varphi}}{1 - \frac{\dot{\varphi}^2}{M^4}} + 3H\dot{\varphi} + M^4(\log V)_\varphi = 0. \quad (4)$$

The energy and pressure densities of the tachyon are:

$$\rho_{\text{T}} = \frac{V(\varphi)}{\sqrt{1 - \frac{\dot{\varphi}^2}{M^4}}}, \quad (5)$$

$$p_{\text{T}} = -V(\varphi)\sqrt{1 - \frac{\dot{\varphi}^2}{M^4}}. \quad (6)$$

The tachyon fluid is also characterized by the ratio between pressure and energy (the equation of state) w_{T} and sound speed c_{T}^2 :

$$w_{\text{T}} = -1 + \frac{\dot{\varphi}^2}{M^4}, \quad c_{\text{T}}^2 = -w_{\text{T}}. \quad (7)$$

Since the equation of state is necessarily nonpositive because of the square root in the action (2), the theory is stable—energy and pressure are real, and inhomogeneous perturbations are equally well behaved. Moreover, because $w_T < 0$, the tachyon is a natural candidate for dark energy and inflation.

Another interesting property is that the equation of state and sound speed of tachyons are equal, but with opposite signs, irrespective of the form for the potential. Canonical scalar fields, on the other hand, obey the Klein-Gordon equations, hence its fluctuations travel with sound speed equal to unity (in units where $c = 1$). Therefore, tachyon fluctuations are fundamentally different from the fluctuations in a canonical scalar field, irrespective of the shape of the potential.

3. TACHYON-DOMINATED BACKGROUNDS

First, we study the evolution of the tachyon as the only component of the universe—i.e., we assume that tachyons are the dominant component. The expansion rate is then given only by the tachyon energy density in Eq. (5):

$$3H^2 = M_{\text{Pl}}^{-2} \frac{V(\varphi)}{\sqrt{1 - \frac{\dot{\varphi}^2}{M^4}}}, \quad (8)$$

where $M_{\text{Pl}}^{-2} = 8\pi G$. Defining the new variables:

$$y \equiv 1 - \frac{\dot{\varphi}^2}{M^4} = -w_T, \quad (9)$$

$$x \equiv V(\varphi), \quad (10)$$

we can reduce the equation of motion (4) to the form:

$$\frac{d \ln y}{dx} = \text{sign}[\dot{\varphi}] y^{-1/4} \sqrt{1 - y} h(x) + \frac{2}{x}, \quad (11)$$

where

$$h(x) \equiv \frac{2\sqrt{3}}{M_{\text{Pl}} M^2} \frac{V^{1/2}}{V_{,\varphi}}. \quad (12)$$

Let us assume now that the tachyon potential is an inverse power-law of the tachyon field:

$$V = m^{4+\alpha} \varphi^{-\alpha}, \quad (13)$$

and also that $\dot{\varphi} > 0$ (exotic initial conditions could be concocted to produce $\dot{\varphi} < 0$ with these potentials, but they are irrelevant to the asymptotic behavior of these models.) The asymptotic behavior of this type of potential is in agreement with the spirit of the original string-inspired proposal (Sen, 2002a,b,c), in which the rolling tachyon field describes the low-energy sector for D-branes and open

strings. Its potential should go to zero at infinite values of the field, such that in this asymptotic vacuum there would be no D-branes, and thus no open strings. Originally the potential was determined to be exponential (Sen, 2002a,b,c). This particular potential can be obtained from the power-law ansatz for the potential by taking appropriately the limit $\alpha \rightarrow \infty$.

Assuming a potential of the form (13) we have:

$$h(x) = -\frac{2\sqrt{3}}{\alpha} M_{\text{pl}}^{-1} M^{-2} m^{\frac{4+\alpha}{\alpha}} x^{-\frac{2+\alpha}{2\alpha}}, \tag{14}$$

which allows us to rewrite Eq. (11) as:

$$\frac{d \ln y}{dx} = -\beta y^{-1/4} \sqrt{1-y} x^{-\frac{2+\alpha}{2\alpha}} + \frac{2}{x}, \tag{15}$$

where the constant $\beta = 2\sqrt{3}\alpha^{-1} M_{\text{pl}}^{-1} M^{-2} m^{(4+\alpha)/\alpha}$.

For $\alpha = 2$ this equation admits an exact solution with $y = \text{constant}$ (Bagla *et al.*, 2003; Feinstein, 2002; Padmanabhan, 2002). This corresponds to a power-law evolution of the scale factor $a(t) = a_0 t^p$ with:

$$p = \frac{1}{3} \left(1 + \sqrt{1 + \frac{9}{4} \frac{m^{12}}{M^8 M_{\text{pl}}^4}} \right), \tag{16}$$

$$\varphi(t) = \sqrt{\frac{2}{3p}} M^2 t. \tag{17}$$

For $\alpha \neq 2$ the system (15) has an attractor as $x \rightarrow 0$ either at $y = 0$ (dust) or at $y = 1$ (quasi-de Sitter), respectively when α is greater or smaller than 2.

The dust solution close to the point ($y \rightarrow 0, x \rightarrow 0$) can be calculated from Eq. (15) by assuming the term inside the square root to be close to unity. We then have the approximate solution when $x \rightarrow 0$:

$$y \simeq \gamma x^s, \tag{18}$$

where

$$s = \frac{2(\alpha - 2)}{\alpha}, \tag{19}$$

$$\gamma = \frac{9}{16} \frac{m^{\frac{4(4+\alpha)}{\alpha}}}{M_{\text{pl}}^4 M^8}. \tag{20}$$

The condition under which this solution satisfies the assumption that $y \rightarrow 0$ as $x \rightarrow 0$ is that $\alpha > 2$. Therefore, for powers of the tachyon potential which are bigger than 2, the late-time behavior of the system is that of a dust-dominated universe. In particular, for $\alpha \rightarrow \infty$ the potential becomes exponential and one

obtains that the dust solution is approached exponentially fast (Frolov *et al.*, 2002).

We now prove that this dust-like solution is stable. A small fluctuation $\delta y(x)$ around the solution (18), using Eq. (15), obeys:

$$\frac{d \ln \delta y}{dx} = \frac{2\alpha - 3}{\alpha} \frac{1}{x}, \tag{21}$$

and therefore the deviation from the dust solution decays as

$$\frac{\delta y}{y} \sim x^{1/\alpha}. \tag{22}$$

The quasi-de Sitter solution, for which $y \rightarrow 1$ as $x \rightarrow 0$, can be obtained by matching the terms in the R.H.S. of Eq. (15), and then improving this solution. The improved solution is:

$$y \simeq 1 - \frac{M_{\text{pl}}^2 M^4}{3m^6} \left(\frac{x}{m^4}\right)^{-1+2/\alpha} \left[1 - \frac{3\alpha - 4}{\alpha} \frac{M_{\text{pl}}^2 M^4}{12m^6} \left(\frac{x}{m^4}\right)^{-1+2/\alpha} \right]. \tag{23}$$

The first term decays when $\alpha < 2$, which means that this solution approaches de Sitter as $x \rightarrow 0$. The last term in the solution above is a further correction that decays even faster. Therefore, the quasi-de Sitter solution is also stable against small fluctuations.

Since the type of acceleration one obtains is perhaps of a peculiar sort, we explicitly derive it. From Eq. (23) one obtains that $\dot{\varphi} \propto \varphi^{1-2/\alpha}$, and substituting this into Eq. (8) one obtains, for the leading term:

$$H = H_0 \left(\frac{t}{t_0}\right)^{-\frac{\alpha}{4-\alpha}}, \tag{24}$$

and therefore

$$a(t) = a_0 \exp \left[\left(\frac{t}{t_0}\right)^{\frac{4-2\alpha}{4-\alpha}} \right]. \tag{25}$$

For $\alpha \rightarrow 0$ one obtains exact de Sitter with $3H^2 = m^4/M_{\text{pl}}^2$. Indeed, the limit $\alpha \rightarrow 0$ corresponds to the so-called Chaplygin Gas (Bento *et al.*, 2002; Bilic *et al.*, 2002; Jackiw, 2000; Kamenshchik *et al.*, 2001), which develops into de Sitter after a period of dust-like behavior obtained by appropriately handling initial conditions. For $\alpha \rightarrow 2$ one obtains from (25) a power-law expansion, in agreement, with the first case discussed in this Section.

Therefore, for inverse power-law tachyon potentials $V \sim \varphi^{-\alpha}$, the asymptotic behavior is very simple: a quasi de Sitter spacetime if $0 < \alpha < 2$, a power-law expansion if $\alpha = 2$, or a dust-like Universe if $\alpha > 2$. The asymptotic tachyon dynamics is summarized in Fig. 1.

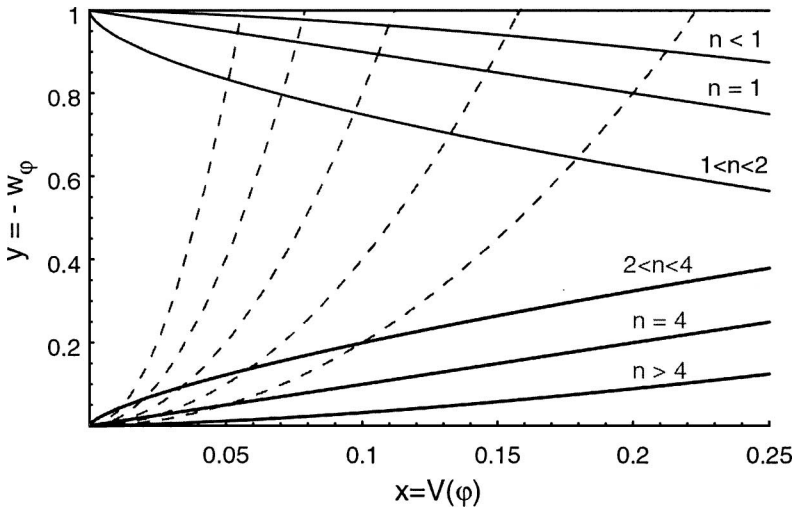


Fig. 1. Phase diagram for the tachyon dynamics.

As already emphasized, a component with nonpositive pressure is interesting for inflation. Inflation driven by a tachyon field has been promptly criticized (Kafman and Linde, 2002) since the effective 4D field theory derived from string theory would lead to a too high scale of inflation (related to m), and thus produce an unacceptable background of stochastic gravitational waves. Hopefully future developments of string theory will help resolve this issue.

For negative values of α , the fate of the tachyon is to oscillate around the minimum of the potential with an averaged equation of state which is negative (Frolov *et al.*, 2002). That the tachyon energy density $\rho_T = x/\sqrt{y}$ is well behaved at the minimum of the potential can be seen from considering the solution $y \propto x^2$ which is found by neglecting the first term in the R.H.S. of Eq. (15) as $x \rightarrow 0$. But if $y \propto x^2$, then the neglected term goes as $x^{-1-1/\alpha}$, which means that it is subdominant with respect to the second term when $\alpha < 0$. However, notice that by Eq. (3) small fluctuations of the tachyon field have an effective mass squared equal to:

$$m_{\text{eff}}^2 = M^4(\ln V)_{,\phi\phi} = \alpha \frac{M^4}{\phi^2}. \tag{26}$$

This means that for negative values of α the tachyon field is unstable under small perturbations (Frolov *et al.*, 2002). Here we focus on *positive* values of α , for which the fluctuations have a positive effective mass.

Since the tachyon energy density decays always slower than that of dust, in the absence of a cosmological constant it is bound to eventually dominate over

the other components (matter and radiation), and it is therefore a natural candidate for inflation and dark energy. Notice that tachyon fluctuations have a sound speed equal to $c_T^2 = -w_T$, and hence have an increased tendency to cluster on sub-Hubble scales compared to standard quintessence (Abramo *et al.*, in preparation). However, if it is to serve as a viable candidate for dark energy, it must have been subdominant in the past. In the next section we analyze the tachyon dynamics in the presence of a perfect fluid-dominated background.

4. COSMOLOGICAL PERTURBATIONS

We discuss gravitational fluctuations for the case in which tachyon is the only component of the universe. Gravitational waves just feel the evolution of the scale factor and in the long-wavelength limit the tensor amplitude h_k behave as:

$$h_k = A_k + B_k \int \frac{dt}{a^3} \tag{27}$$

where A_k and B_k are functions of k . Besides the constant mode A_k , the solution containing B_k is decaying for the type of inflationary and dust solutions discussed in this paper.

Scalar metric fluctuations are described by the Mukhanov variable v whose evolution is (Garriga and Mukhanov, 1999)

$$v_k'' + \left(c_T^2 k^2 - \frac{z''}{z} \right) v_k = 0, \tag{28}$$

where ' denotes derivative with respect to the conformal time ($d\eta = dt/a$), and (Frolov *et al.*, 2002):

$$z = a \frac{\sqrt{\rho_T + p_T}}{c_T H} = a \sqrt{3} \frac{M_{Pl}}{M^2} \frac{\dot{\phi}}{\sqrt{1 - \frac{\dot{\phi}^2}{M^4}}}. \tag{29}$$

The solution for v_k in the long-wavelength limit is:

$$v_k = C_k z + D_k z \int \frac{dt}{az^2}, \tag{30}$$

where C_k and D_k are functions of k . The curvature perturbation $\zeta \equiv v/z$ in the long-wavelength limit is therefore:

$$\zeta_k = C_k + D_k \int \frac{dt}{az^2}. \tag{31}$$

Besides the constant mode, the solution containing D_k is decaying for the type of inflationary and dust solutions discussed in this paper.

The curvature perturbation ζ remains constant on large scales, even if the gauge invariant field fluctuation $\delta\varphi = v/a$ can vary in time:

$$\delta\varphi_k \propto \frac{\dot{\varphi}}{c_T}. \quad (32)$$

More precisely, on large scales, tachyon fluctuations decrease during inflation and grow in the dust regime.

In the dust regime metric perturbations remain constant. In fact, even the Newtonian potential Φ , which satisfy:

$$\zeta = \frac{2}{3} \frac{\dot{\Phi}/H + \Phi}{1 + w_T} + \Phi \quad (33)$$

remains constant on long-wavelength scales, satisfying the usual relation $\Phi \simeq 3/5\zeta$ of matter-dominated era. However, long-wavelength field fluctuations build up in time as:

$$\delta\varphi \sim t^{\alpha-1} \rightarrow \delta\varphi/\varphi \sim t^{\alpha-2} \quad (34)$$

leading to a nonlinear stage for the tachyon field. Metric fluctuations are however kept constant by a compensating term. These considerations hold when the tachyon behaves as dust, irrespective of the form of the potential. Since at the level of linear perturbation theory the metric fluctuations stay constant even as the field fluctuations grow on large scales, it remains unproved that metric fluctuations at large scales will grow at the nonlinear level.

We also would like to emphasize that the effective mass for tachyon fluctuations in Eq. (26) is really different from the scale of inflation. This is attractive if one thinks at the criticism about the flatness of the potential (Adams *et al.*, 1991). In the spirit of k -inflation (Garriga and Mukhanov, 1999), the prediction about the ratio of scalar to tensor perturbations will be different from the one of canonical scalar field models. So, in principle inflation with the tachyon can be distinguished from canonical scalar field inflation.

5. TACHYON AND PERFECT FLUID DYNAMICS

We now study the evolution of the tachyon fluid in the background of a perfect fluid characterized by energy density ρ_F and constant equation of state w_F . The Hubble law is therefore:

$$H^2 = \frac{1}{3M_{\text{Pl}}^2} \left[\frac{V(\varphi)}{\sqrt{1 - \frac{\dot{\varphi}^2}{M^4}}} + \rho_F \right] \quad (35)$$

The equation of motion for the tachyon (4) can be satisfied for $\dot{\varphi} = \text{constant}$. This corresponds to a perfect fluid-like evolution for the tachyon energy density. The

particular case $\dot{\varphi} = M^2 (w_T = 0)$ is interesting for the tachyon as a candidate for CDM. However, other fluid-like attractors exist for the tachyon with a negative power-law potential (13) in the presence of another dominating fluid if $0 < \alpha < 2$.

If the tachyon component is subdominant with respect to the background fluid, Eq. (35) means that:

$$H \simeq \frac{2}{3(1 + w_F)t}. \tag{36}$$

With the potential (13) a solution for the homogeneous tachyon field in which its amplitude grows linearly in time is possible:

$$\varphi \simeq At \quad \text{with} \quad A = M^2 \sqrt{\alpha \frac{1 + w_F}{2}}, \tag{37}$$

where we have neglected the initial value φ_0 for the tachyon and a negative sign for the tachyon velocity (a tachyon climbing up its effective potential.) Neglecting the initial value translates in an inequality for the growing tachyon:

$$\varphi_0 H_0 \ll M^2 \sqrt{\frac{2\alpha}{9(1 + w_F)}} \tag{38}$$

where the subscript 0 denotes some primordial time.

Small deviations $\delta y_A(t)$ around the tracking solution (37) satisfy the following equation:

$$\delta \ddot{y}_A + \left(\frac{2}{1 + w_F} - \alpha \right) \frac{\delta \dot{y}_A}{t} + \left(\frac{2}{1 + w_F} - \alpha \right) \frac{\delta y_A}{t^2} = 0. \tag{39}$$

The solutions are of power-law type, i.e. $\delta y_A \sim t^{\gamma_A}$, with:

$$\gamma_A = \frac{1}{2}(1 + \lambda) \pm \frac{1}{2}\sqrt{(1 + \lambda)^2 + 4\lambda}, \tag{40}$$

where $\lambda = \alpha - 2/(1 + w_F)$. This shows that the tracking solutions (37) are stable for $\lambda < 0$, i.e., $\alpha < 2/(1 + w_F)$.

For this class of solution the tachyon field has an equation of state:

$$w_T = -1 + \frac{A^2}{M^4} = -1 + \alpha \frac{1 + w_F}{2}. \tag{41}$$

However, these solutions can only exist if they respect the condition that the tachyon equation of state is nonpositive. This means that, for instance, the fluid-like solution is only valid in the range $0 < \alpha < 3/2$ if the background is radiation ($w_F = 1/3$).³

³ In Chiba (2002) it was argued that $0 < \alpha < 2$ for the general case in which the Lagrangian is separable: $\mathcal{L} = p(\varphi, X) = V(\phi)W(X)$, where $X = \partial^\mu \varphi \partial_\mu \varphi$. We obtain a different value because we do not assume $w_F = 0$ (dust) for the tracking.

For the range $2/(1 + w_F) < \alpha < 2$ the attractor saturates the dust limit—i.e., $w_T \rightarrow 0^-$. The solution of Eq. (4) in that case is given by:

$$\varphi = M^2 t - \epsilon t^{1-2(\alpha-\frac{2}{1+w_F})}, \quad (42)$$

where ϵ is an arbitrary constant that depends on the initial conditions. The correction term decays in time compared to the leading term if $\alpha > 2/(1 + w_F)$, showing that dust is indeed an attractor for this range of parameters.

A curious property of tachyons as candidates for dark energy is that when they are in a background which is dominated by a fluid with equation of state w_F , their energy density behaves either as $\rho_T \propto a^{-3}$ when $\alpha > 2/(1 + w_F)$, or as $\rho_T \propto t^{-\alpha}$ if $0 < \alpha < 2/(1 + w_F)$. But on the other hand, when the tachyon starts to dominate, it will behave as dust if $\alpha > 2$ or it will drive quasi-de Sitter acceleration if $0 < \alpha < 2$. Therefore, if $2/(1 + w_F) < \alpha < 2$, a change in the equation of state of the background (as, for example, the change from radiation- to matter-domination) will trigger a transmutation in the tachyon, causing it to leave the dust attractor and start to accelerate the expansion rate (a similar behavior was found in the “*k*-essence” model (Armendariz-Picon *et al.*, 2000).

6. CONCLUSIONS

We have analyzed the cosmological dynamics of a Born-Infeld scalar field (also known as the tachyon) with an inverse power-law potential. We found that the tachyon has a very simple behavior: if the tachyon dominates the background dynamics, then it will either go into a dust-dominated phase ($\alpha > 2$), power-law expansion with a constant $w_t < 0$ for $\alpha = 2$, or quasi-de Sitter accelerated expansion for $0 < \alpha < 2$.

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